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Effective Potential for D-brane in Constant Electromagnetic Field

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ABSTRACT

We discuss the one-loop effective potential and static (large d) potential for toroidal D-brane described by DBI-action in constant magnetic and in constant electric fields. Explicit calculation is done for membrane case ($p = 2$) for both types of external fields and in case of static potential for an arbitrary p . In the case of one-loop potential it is found that the presence of magnetic background may stabilize D-brane giving the possibility for non-pointlike ground state configuration. On the same time, constant electrical field acts against stabilization and the correspondent one-loop potential is unbounded from below. The properties of static potential which also has stable minimum are described in detail. The back-reaction of quantum gauge fields to one-loop potential is also evaluated.

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1 Introduction

There was recently much of interest in the study of the dynamics of $D = 11$ M -theory [1]. One of the key ingredients of M -theory is D-brane [2, 3] which describes the non-perturbative dynamics of string theory [4] and gives new insight into understanding of stringy black holes [5].

D-brane theories are described by Born-Infeld(BI)-type actions where the world volume vector field naturally appears. Such theories have been the subject of much recent work [6]. Note that BI- type effective action of open string theory has been discussed in refs.[7].

It is well-known that some physical properties of the extended objects in the quantum regime may be gleaned from a study of the effective action for various static p -brane configurations. In this way one studies the effective action in one-loop or in large d approximation. For string case, large d approximation has been developed in ref.[8] as systematic expansion for the effective action in powers of $1/d$. The static potential may be then obtained by studying the saddle point equations for the leading order term. Such program has been realized also for rigid string [9], rigid string at non-zero temperature [10] (where it is expected to be helpful in drawing the relations between QCD and string theory), bosonic membrane [11, 12] and bosonic p -brane [12].

The purpose of present work is to investigate the one-loop and static potentials for D-brane in the background gauge fields. We start such calculation in the next section for situation when external electromagnetic fields are purely classical. The one-loop potential in the constant magnetic and in the constant electric field is found for $p = 2$ (membrane). We show that unlike the case without electromagnetic field there exists non-trivial minimum of effective potential corresponding to the non-pointlike ground state. The static potential in the same background is also calculated and its properties are discussed. Third section is devoted to the study of back reaction of quantum gauge fields to effective potential. We calculate the one-loop potential in the same background with additional contribution due to quantum gauge fields but it is found that the qualitative natures of the obtained one-loop potential are not changed. Some remarks and outline are given in Discussion.

2 One-loop and static potential for D-brane in constant electromagnetic field.

The D-brane is described by the Dirac-Born-Infeld-type action [6]

$$S_D = k \int_0^T d\zeta_0 \int d^p \zeta e^{-\phi(X)} [\det ((G_{\mu\nu} + B_{\mu\nu}) \partial_i X^\mu \partial_j X^\nu + F_{ij})]^{\frac{1}{2}} . \quad (1)$$

Here X^μ 's are the coordinates of D-brane ($\mu, \nu = 0, 1, \dots, d-1$), ζ^i 's are the coordinates on the D-brane world sheet ($i, j = 0, 1, \dots, p$), $G_{\mu\nu}$ is the metric of the space-time, $B_{\mu\nu}$ is the anti-symmetric tensor and F_{ij} is the electromagnetic field strength on the D-brane world sheet:

$$F_{ij} = \partial_i A_j - \partial_j A_i . \quad (2)$$

We are now interested in the stability of the D-brane and study the effective potential. The effective potentials of p -brane were studied in [12]. In this paper, we investigate the case that F_{ij} (or B_{ij}) has nontrivial vacuum expectation value.

If we choose the following background gauge choice

$$X^i = R_i \zeta^i \quad (R_0 = 1) \quad i = 0, 1, \dots, p \quad (3)$$

and impose the periodic boundary conditions corresponding to the toroidal D-brane

$$\begin{aligned} X^m(\zeta^0, \zeta^1, \zeta^2, \dots, \zeta^p) &= X^m(\zeta^0 + T, \zeta^1, \zeta^2, \dots, \zeta^p) \\ &= X^m(\zeta^0, \zeta^1 + 1, \zeta^2, \dots, \zeta^p) = X^m(\zeta^0, \zeta^1, \zeta^2 + 1, \dots, \zeta^p) \\ &= \dots = X^m(\zeta^0, \zeta^1, \zeta^2, \dots, \zeta^p + 1) \\ &\quad (m = p+1, p+2, \dots, d-1) , \end{aligned} \quad (4)$$

we obtain the following gauge-fixed action,

$$\begin{aligned} S_D &= k \int_0^T d\zeta_0 \int d^p \zeta e^{-\phi(X)} \\ &\quad \times \left[\det \left(\hat{G}_{ij} + \hat{G}_{im} \partial_j X^m + \hat{G}_{mj} \partial_i X^m + G_{mn} \partial_i X^m \partial_j X^n \right) \right]^{\frac{1}{2}} . \end{aligned} \quad (5)$$

Here

$$\begin{aligned}
\hat{G}_{ij} &= R_i R_j (G_{ij} + B_{ij}) + F_{ij} \\
\hat{G}_{im} &= R_i (G_{im} + B_{im}) \\
\hat{G}_{im} &= R_i (G_{im} + B_{im}) \\
\hat{G}_{mn} &= G_{mn} + B_{mn} .
\end{aligned} \tag{6}$$

Note that in the above gauge there are no Faddeev-Popov ghosts. For simplicity, we only consider the case

$$\begin{aligned}
\phi &= 0 \\
G_{\mu\nu} &= \delta_{\mu\nu} \\
B_{mn} &= B_{im} = B_{mi} = 0 .
\end{aligned} \tag{7}$$

Then we obtain,

$$S_D = k \int_0^T d\zeta_0 \int d^p \zeta \left[\det \left(\hat{G}_{ij} + \partial_i X^\perp \cdot \partial_j X^\perp \right) \right]^{\frac{1}{2}} \tag{8}$$

and the effective potential is defined by

$$V = - \lim_{T \rightarrow \infty} \frac{1}{T} \ln \int \frac{DX^\perp DA_i}{V_A} e^{-S_D} . \tag{9}$$

Here V_A is the gauge volume for the gauge field A_i and

$$X^\perp = (X^{p+1}, X^{p+2}, \dots, X^{d-1}) \tag{10}$$

$$\begin{aligned}
\hat{G}_{ij} &= R_i R_j \delta_{ij} + \mathcal{F}_{ij} \\
\mathcal{F}_{ij} &\equiv R_i R_j B_{ij} + F_{ij} .
\end{aligned} \tag{11}$$

As we can see from (11), the anti-symmetric part \mathcal{F}_{ij} of \hat{G}_{ij} contains the contribution from the anti-symmetric tensor B_{ij} and the gauge field on the D-brane world sheet. For a while, we treat the electromagnetic field strength F_{ij} as classical field and assume \mathcal{F}_{ij} has a constant and nontrivial vacuum expectation value.

We now define the following tensors

$$\begin{aligned}
G^{Sij} &\equiv \frac{1}{2} \left((\hat{G}^{-1})^{ij} + (\hat{G}^{-1})^{ji} \right) \\
\tilde{G}^{\alpha\beta} &\equiv \frac{1}{G^{S00}} \left(G^{S\alpha\beta} - \frac{1}{G^{S00}} G^{S0\alpha} G^{S0\beta} \right)
\end{aligned} \tag{12}$$

Here $(\hat{G}^{-1})^{ij}$ is the inverse matrix of \hat{G}_{ij} and $\alpha, \beta = 1, 2, \dots, p$. Then the one-loop effective potential is given by (compare with [12])

$$V_T = k(\det \hat{G}_{ij})^{\frac{1}{2}} + \frac{1}{2}(d - p - 1) \sum_{n_1, n_2, \dots, n_p = -\infty}^{\infty} \left(4\pi^2 \sum_{k, l=1}^p \tilde{G}^{kl} n_k n_l \right)^{\frac{1}{2}}. \quad (13)$$

In the following, we consider some examples for the choice of electromagnetic background.

First we consider the membrane ($p = 2$) with the magnetic background

$$\mathcal{F}_{0k} = 0, \quad \mathcal{F}_{12} = -\mathcal{F}_{21} = h \quad (14)$$

and assume

$$R_1 = R_2 = R. \quad (15)$$

Then we obtain

$$\hat{G} \equiv \det \hat{G}_{ij} = R^4 + h^2 \quad (16)$$

$$(\hat{G}^{-1})^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \hat{G}^{-1} R^2 & -\hat{G}^{-1} h \\ 0 & \hat{G}^{-1} h & \hat{G}^{-1} R^2 \end{pmatrix} \quad (17)$$

$$\tilde{G}^{\alpha\beta} = \begin{pmatrix} \hat{G}^{-1} R^2 & 0 \\ 0 & \hat{G}^{-1} R^2 \end{pmatrix} \quad (18)$$

and we find that the one-loop potential has the following form

$$V_T = k(R^4 + h^2)^{\frac{1}{2}} + \frac{d-3}{2} \cdot \frac{R}{(R^4 + h^2)^{\frac{1}{2}}} f_T(1, 1). \quad (19)$$

Here we have used zeta-function regularization [13] and we obtain [12]

$$f_T(1, 1) = 2\pi \sum_{n_1, n_2 = -\infty}^{\infty} (n_1^2 + n_2^2)^{\frac{1}{2}} = -1.438 \dots \quad (20)$$

If we assume the magnetic flux

$$\Phi = \int d\zeta_1 d\zeta_2 h \quad (21)$$

(note that \mathcal{F}_{ij} is a two-form) is size-independent, h does not depend on R . Contrary to $h = 0$ case in [12], the effective potential (19) has a non-trivial minimum since $f_T(1, 1) < 0$. Hence, unlike the case $h = 0$ toroidal D-brane will not tend to collapse. There exists non-pointlike ground state with finite radius. (The explicit value of this radius is very complicated). This ground state is stabilized due to external magnetic field effects.

In case that D-brane has open boundaries, we obtain the one-loop potential similar to (19). The potential of the open D-brane has a non-trivial relative minimum besides $R = 0$ when k is small but $R = 0$ becomes a stable minimum since $V_T(R > 0) > V_T(R = 0)$.

We also consider the one-loop potential of the membrane in the constant electric background where

$$\mathcal{F}_{0k} = e, \quad \mathcal{F}_{kl} = 0, \quad R_1 = R_2 = R. \quad (22)$$

Then we find

$$\hat{G} \equiv \det \hat{G}_{ij} = R^4 + 2e^2 R^2 \quad (23)$$

$$(\hat{G}^{-1})^{ij} = \hat{G}^{-1} \begin{pmatrix} R^4 & -eR^2 & -eR^2 \\ eR^2 & R^2 + e^2 & -e^2 \\ eR^2 & -e^2 & R^2 + e^2 \end{pmatrix} \quad (24)$$

$$\tilde{G}^{kl} = \begin{pmatrix} R^2 + e^2 & -e^2 \\ -e^2 & R^2 + e^2 \end{pmatrix}. \quad (25)$$

The total one-loop potential is given by

$$V_T = k(R^4 + 2e^2)^{\frac{1}{2}} + \frac{d-3}{2} \cdot \frac{(R^2 + e^2)^{\frac{1}{2}}}{R^2} \hat{f}_T(1, 1, -\frac{e^2}{R^2 + e^2}). \quad (26)$$

Here $\hat{f}_T(1, 1, s)$ is defined by using of modified Bessel function K_ν [13]

$$\begin{aligned} \hat{f}_T(1, 1, s) &\equiv 2\pi \sum_{n_1, n_2 = -\infty}^{\infty} \left(n_1^2 + n_2^2 + 2sn_1n_2 \right)^{\frac{1}{2}} \\ &= 4\pi\zeta(-1) - \frac{\zeta(3)}{2\pi} \Delta \\ &\quad - 4\Delta^{\frac{1}{2}} \sum_{n=1}^{\infty} \frac{\sigma_2(n) \cos(2sn\pi)}{n} K_1(\pi n \Delta^{\frac{1}{2}}) \end{aligned} \quad (27)$$

and

$$\Delta \equiv 4 - 4s^2, \quad \sigma_s(n) \equiv \sum_{d|n} d^s. \quad (28)$$

In order to discuss the stability of the one-loop potential in Eq.(26), we consider the behavior when $R \rightarrow 0$, i.e., $s \rightarrow -1$ and $\Delta \sim \frac{4R^2}{e^2} \rightarrow 0$ in Eq.(27). The asymptotic behavior of $\sigma_2(n)$ when $n \rightarrow \infty$ is given by,

$$\sigma_2(n) = n^2 + \mathcal{O}(n). \quad (29)$$

By using the expression for $K_1(z)$

$$K_1(z) = z \int_1^\infty dt e^{-zt} (t^2 - 1)^{\frac{1}{2}} \quad (30)$$

and Eq.(29), we find that the behavior of the third term in the r.h.s. of Eq.(27) when $R \rightarrow 0$ is given by

$$\begin{aligned} & -4\Delta^{\frac{1}{2}} \sum_{n=1}^\infty \frac{\sigma_2(n) \cos(2sn\pi)}{n} K_1(\pi n \Delta^{\frac{1}{2}}) \\ & \sim -4\pi\Delta \int_1^\infty dt (t^2 - 1)^{\frac{1}{2}} \sum_{n=1}^\infty n^2 \left(e^{-\pi\Delta^{\frac{1}{2}}t} \right)^n \\ & \sim -4\pi\Delta \int_1^\infty dt \frac{(t^2 - 1)^{\frac{1}{2}}}{\left(1 - e^{-\pi\Delta^{\frac{1}{2}}t} \right)^3} \\ & \sim -4\pi^{-2} c_0 \Delta^{-\frac{1}{2}} \\ & \sim -\frac{2c_0 e}{\pi^2 R}. \end{aligned} \quad (31)$$

Here

$$c_0 \equiv \int_1^\infty dt \frac{(t^2 - 1)^{\frac{1}{2}}}{t^3}. \quad (32)$$

Eq.(31) tells that the potential (26) appears to be unstable since it is unbounded below when $R \rightarrow 0$. It is not strange because we expect that constant electrical field (unlike the magnetic field) leads to pairs creation and it should quickly destabilize the static approximation. However, when non-perturbative effects are taken into account (as we will see later for static potential in the same background) they play more essential role near $R = 0$

than destabilizing effect of the electrical field. As a result static potential has stable minimum even in the electrical background.

If h in (14) and e in (22) come from B_{ij} in (11), they can be considered to be pure background fields and we need not to vary them to find the saddle point. Note that, however, they become size-dependent as found in (11) and we should replace them by

$$h \rightarrow R^2 h, \quad e \rightarrow R e. \quad (33)$$

The new parameters h and e are size-independent.

Even when h and e come from F_{ij} in (11), we should not vary them since their flux would be conserved. If we assume the flux is conserved, h and e should be size-independent.

We can also consider the static potential in the limit of $d \rightarrow \infty$. By introducing the auxiliary fields σ_{ij} and λ^{ij} , the action in (5) is rewritten as

$$\begin{aligned} S_D = & k \int_0^T d\zeta_0 \int d^p \zeta \left[\left\{ \det (\hat{G}_{ij} + \sigma_{ij}) \right\}^{\frac{1}{2}} \right. \\ & \left. - \frac{k}{2} \lambda^{ij} (\partial_i X^\perp \cdot \partial_j X^\perp - \sigma_{ij}) \right]. \end{aligned} \quad (34)$$

By integrating X^\perp we obtain the large- d effective action

$$\begin{aligned} S_{eff} = & \frac{1}{2} (d - p - 1) \text{Tr} \ln (-\partial_i \lambda^{ij} \partial_j) \\ & + kT \left[\left\{ \det (\hat{G}_{ij} + \sigma_{ij}) \right\}^{\frac{1}{2}} - \frac{1}{2} \text{Tr} (\lambda \cdot \sigma) \right] \end{aligned} \quad (35)$$

In the large d limit, we can only consider the saddle point and the auxiliary fields σ_{ij} and λ^{ij} can be treated as classical fields.

We now consider the D-brane with magnetic field background when p is even

$$f_{2n-1\,2n} = h, \quad n = 1, 2, \dots, \frac{p}{2} \quad (36)$$

in the large d approximation. We assume

$$R_i = R \quad (37)$$

and the auxiliary fields σ_{ij} and λ^{ij} have the following form in the saddle point

$$\begin{aligned} \sigma_{ij} &= \text{diag}(\sigma_0, \sigma_1, \sigma_1) + \mathcal{O}(1/d) \\ \lambda_{ij} &= \text{diag}(\lambda_0, \lambda_1, \lambda_1) + \mathcal{O}(1/d). \end{aligned} \quad (38)$$

Since $\det(\hat{G}_{ij} + \sigma_{ij})$ in Eq.(35) is given by

$$\det(\hat{G}_{ij} + \sigma_{ij}) = (1 + \sigma_0) \left\{ (R^2 + \sigma_1)^2 + h^2 \right\}^{\frac{p}{2}}, \quad (39)$$

we find that the effective action has the following form

$$\begin{aligned} S_{eff} = & kT \left[(1 + \sigma_0)^{\frac{1}{2}} \{ (R^2 + \sigma_1)^2 + h^2 \}^{\frac{p}{4}} \right. \\ & \left. - \frac{1}{2} R^2 (\sigma_0 \lambda_0 + 2\sigma_1 \lambda_1) + R^2 \alpha \left(\frac{\lambda_1}{\lambda_0} \right)^{\frac{1}{2}} \right] \end{aligned} \quad (40)$$

where α is defined by

$$\alpha \equiv \frac{d - p - 2}{2kR^{p+1}} f_T(1, 1, \dots, 1) \quad (41)$$

$$f_T(1, 1, \dots, 1) \equiv 2\pi \sum_{n_1, n_2, \dots, n_p = -\infty}^{\infty} \left(\sum_{i=1}^p n_i^2 \right)^{\frac{1}{2}}. \quad (42)$$

The variations with respect to σ_0 , λ_0 and λ_1 give the following equations, respectively:

$$R^p \lambda_0 = (1 + \sigma_0)^{-\frac{1}{2}} \{ (R^2 + \sigma_1)^2 + h^2 \}^{\frac{p}{4}} \quad (43)$$

$$\sigma_0 = -\alpha \lambda_1^{\frac{1}{2}} \lambda_0^{-\frac{3}{2}} \quad (44)$$

$$p\sigma_1 = \alpha \lambda_1^{-\frac{1}{2}} \lambda_0^{-\frac{1}{2}}. \quad (45)$$

We now solve σ_0 , λ_0 , and λ_1 with respect to σ_1 by using Eqs.(43), (44) and (45).

By using (44) and (45), we obtain

$$p\sigma_0\sigma_1 = -\alpha^2 \lambda_0^{-2} \quad (46)$$

$$\frac{\lambda_1}{\lambda_0} = -\frac{1}{p} \frac{\sigma_0}{\sigma_1} \quad (47)$$

$$\sigma_0 \lambda_0 + p\sigma_1 \lambda_1 = 0. \quad (48)$$

By using (43) and (46), we can solve for σ_0 :

$$\sigma_0 = -\frac{\alpha^2 R^{2p}}{\alpha^2 R^{2p} + p\sigma_1 \{ (R^2 + \sigma_1)^2 + h^2 \}^{\frac{p}{2}}}. \quad (49)$$

Then we find

$$\begin{aligned}
S_{eff} &= kT \left[\frac{\sqrt{p}}{|\alpha|R^p} \{ (R^2 + \sigma_1)^2 + h^2 \} (-\sigma_0)^{\frac{1}{2}} \sigma_1^{\frac{1}{2}} + \frac{1}{\sqrt{p}} R^p |\alpha| \frac{(-\sigma_0)^{\frac{1}{2}}}{\sigma_1^{\frac{1}{2}}} \right] \\
&= \frac{kT}{\sqrt{p}} \sigma_1^{-\frac{1}{2}} \left[2 \{ (R^2 + \sigma_1)^2 + h^2 \}^{\frac{p}{2}} \sigma_1 + \alpha^2 R^{2p} \right]^{\frac{1}{2}}
\end{aligned} \tag{50}$$

and the static potential V_T with respect to σ_1 is given by

$$V_T = \frac{k}{\sqrt{p}} \sigma_1^{-\frac{1}{2}} \left[2 \{ (R^2 + \sigma_1)^2 + h^2 \}^{\frac{p}{2}} \sigma_1 + \alpha^2 R^{2p} \right]^{\frac{1}{2}} . \tag{51}$$

Note that the static potential V is defined by

$$V \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \cdot S_{eff} . \tag{52}$$

We can easily find that there exists a stable non-trivial minimum in Eq.(51). The minimum exists even when $h = 0$ in [12].

The large d effective potential in the electric background:

$$\mathcal{F}_{0k} = e_k , \quad \mathcal{F}_{kl} = 0 , \quad R_k = R \tag{53}$$

can be found similarly. Since we find

$$\det(\hat{G}_{ij} + \sigma_{ij}) = (1 + \sigma_0)(R^2 + \sigma_1)^p + e^2(R^2 + \sigma_1)^{p-1} \tag{54}$$

where

$$e^2 \equiv \sum_{k=1}^p e_k^2 . \tag{55}$$

The effective action is given by

$$\begin{aligned}
S_{eff} &= kT \left[\{ (1 + \sigma_0)(R^2 + \sigma_1)^p + e^2(R^2 + \sigma_1)^{p-1} \}^{\frac{1}{2}} \right. \\
&\quad \left. - \frac{1}{2} R^p (\sigma_0 \lambda_0 + p \sigma_1 \lambda_1) + R^p \alpha \left(\frac{\lambda_1}{\lambda_0} \right)^{\frac{1}{2}} \right]
\end{aligned} \tag{56}$$

Here α is defined in (41). The variations with respect to σ_0 , λ_0 and λ_1 give the following equations, respectively:

$$R^p \lambda_0 = \{ (1 + \sigma_0)(R^2 + \sigma_1)^p + e^2(R^2 + \sigma_1)^{p-1} \}^{-\frac{1}{2}} (R^2 + \sigma_1)^p \tag{57}$$

$$\sigma_0 = -\alpha \lambda_1^{\frac{1}{2}} \lambda_0^{-\frac{3}{2}} \tag{58}$$

$$p \sigma_1 = \alpha \lambda_1^{-\frac{1}{2}} \lambda_0^{-\frac{1}{2}} . \tag{59}$$

By using (58) and (59), we obtain

$$p\sigma_0\sigma_1 = -\alpha^2\lambda_0^{-2} \quad (60)$$

$$\frac{\lambda_1}{\lambda_0} = -\frac{1}{p}\frac{\sigma_0}{\sigma_1} \quad (61)$$

$$\sigma_0\lambda_0 + p\sigma_1\lambda_1 = 0. \quad (62)$$

By using (57) and (60), we can solve for σ_0 :

$$\sigma_0 = -\frac{\alpha^2 R^{2p} \{1 + e^2(R^2 + \sigma_1)^{-1}\}}{\alpha^2 R^{2p} + p\sigma_1(R^2 + \sigma_1)^p}. \quad (63)$$

Then we find

$$\begin{aligned} S_{eff} &= kT \left\{ \frac{\sqrt{p}}{\alpha R^p} (R^2 + \sigma_1)^p (-\sigma_0)^{\frac{1}{2}} \sigma_1^{\frac{1}{2}} + \frac{1}{\sqrt{p}} R^p \alpha \frac{(-\sigma_0)^{\frac{1}{2}}}{\sigma_1^{\frac{1}{2}}} \right\} \\ &= \frac{kT}{\sqrt{p}} \sigma_1^{-\frac{1}{2}} \left\{ 1 + e^2(R^2 + \sigma_1)^{-1} \right\}^{\frac{1}{2}} \left[p(R^2 + \sigma_1)^p \sigma_1 + \alpha^2 R^{2p} \right]^{\frac{1}{2}} \end{aligned} \quad (64)$$

$$V_T = \frac{k}{\sqrt{p}} \sigma_1^{-\frac{1}{2}} \left\{ 1 + e^2(R^2 + \sigma_1)^{-1} \right\}^{\frac{1}{2}} \left[p(R^2 + \sigma_1)^p \sigma_1 + \alpha^2 R^{2p} \right]^{\frac{1}{2}}. \quad (65)$$

The static potential (65) has also only one stable minimum. The minimum also exists when $e = 0$.

Hence, one can see that non-perturbative effects of large d expansion play more essential role than destabilizing effect of the constant electrical field. As a result this potential shows the possibility of non-trivial minimum (non-pointlike ground state).

3 One-loop potential with the quantum back-reaction of electromagnetic field.

In the following, we consider the quantum back-reaction of the gauge field by dividing the anti-symmetric part \mathcal{F}_{ij} in \hat{G}_{ij} into the sum of the classical part \mathcal{F}_{ij}^c and quantum fluctuation F_{ij}^q :

$$\mathcal{F}_{ij} = \mathcal{F}_{ij}^c + F_{ij}^q. \quad (66)$$

In the following, we write $R_i^2 \delta_{ij} + \mathcal{F}_{ij}^c$ as \hat{G}_{ij} and F_{ij}^q as F_{ij} .

We use the one-loop approximation by expanding the action (5) and keeping the quadratic term. Since

$$\frac{\partial \hat{G}}{\partial \hat{G}_{ij}} = \hat{G}(\hat{G}^{-1})^{ji} \quad (67)$$

$$\frac{\partial^2 \hat{G}}{\partial \hat{G}_{ij} \partial \hat{G}_{kl}} = \hat{G} \left\{ -(\hat{G}^{-1})^{jk}(\hat{G}^{-1})^{li} + (\hat{G}^{-1})^{ji}(\hat{G}^{-1})^{lk} \right\} \quad (68)$$

we find

$$\begin{aligned} & \left\{ \det (\hat{G}_{ij} + F_{ij}) \right\}^{\frac{1}{2}} \\ &= \hat{G}^{\frac{1}{2}} \left[1 + \frac{1}{2} (\hat{G}^{-1})^{ji} F_{ij} \right. \\ & \quad \left. + \left\{ -\frac{1}{4} (\hat{G}^{-1})^{jk}(\hat{G}^{-1})^{li} + \frac{1}{8} (\hat{G}^{-1})^{ji}(\hat{G}^{-1})^{lk} \right\} F_{ij} F_{kl} + \cdots \right]. \end{aligned} \quad (69)$$

If \hat{G}_{ij} is a constant tensor, the second term is a total derivative and does not give any contribution.

Let's consider the following easy example, where \hat{G}_{ij} is given by

$$\hat{G}_{ij} = R_i^2 \delta_{ij} \quad (70)$$

Then

$$\left\{ \det (\hat{G}_{ij} + F_{ij}) \right\}^{\frac{1}{2}} = \hat{G}^{\frac{1}{2}} \left[1 - \frac{1}{4} \sum_{i,j=0}^p \frac{1}{R_i^2 R_j^2} F_{ij}^2 + \cdots \right]. \quad (71)$$

Here we neglect total derivative terms. By rescaling the gauge potential

$$A_i \rightarrow R_i A_i \quad (72)$$

($R_0 = 1$), Eq.(71) is rewritten by

$$\begin{aligned} \left\{ \det (\hat{G}_{ij} + F_{ij}) \right\}^{\frac{1}{2}} &= \hat{G}^{\frac{1}{2}} \left[1 + \frac{1}{2} \sum_{i,j=0}^p A_i \left\{ \left(\sum_{k=0}^p \frac{\partial_k^2}{R_k^2} \right) \delta^{ij} - \frac{\partial_i \partial_j}{R_i R_j} \right\} A_j \right. \\ & \quad \left. + \text{total derivative terms} + \cdots \right]. \end{aligned} \quad (73)$$

If we choose the gauge condition

$$A_0 = 0 \quad (74)$$

we find the residual gauge condition (transverse condition)

$$\sum_{i=1}^p \frac{1}{R_i} \partial_i A_i = 0 \quad (75)$$

and we obtain

$$\begin{aligned} & \left\{ \det \left(\hat{G}_{ij} + F_{ij} \right) \right\}^{\frac{1}{2}} \\ &= \hat{G}^{\frac{1}{2}} \left[1 + \frac{1}{2} \sum_{\substack{i : \text{ transverse} \\ \text{components}}} A_i \left(\sum_{k=0}^p \frac{\partial_k^2}{R_k^2} \right) A_i + \cdots \right] . \end{aligned} \quad (76)$$

Then we find that the contribution from the gauge field to the one-loop potential is given by

$$\frac{1}{2}(p-1) \sum_{n_1, n_2, \dots, n_p = -\infty}^{\infty} \left(\sum_{i=1}^p \frac{4\pi^2 n_i^2}{R_i^2} \right)^{\frac{1}{2}} . \quad (77)$$

Therefore the qualitative nature of the one-loop potential does not change if compare to the case without the contribution from the gauge field.

The one-loop potential in the toroidal membrane ($p = 2$) with the magnetic background (14) (including the quantum backreaction of the gauge fields) can be obtained similarly. The kinetic term of the gauge potential is given by

$$\begin{aligned} & \left\{ -\frac{1}{4}(\hat{G}^{-1})^{jk}(\hat{G}^{-1})^{li} + \frac{1}{8}(\hat{G}^{-1})^{ji}(\hat{G}^{-1})^{lk} \right\} F_{ij} F_{kl} \\ &= \frac{1}{2} \hat{G}^{-1} R^2 (F_{01}^2 + F_{02}^2) + \frac{1}{2} \hat{G}^{-2} R^4 F_{12}^2 . \end{aligned} \quad (78)$$

By choosing the gauge fixing condition (74), we obtain the transverse condition

$$\partial_1 A_1 + \partial_2 A_2 = 0 . \quad (79)$$

By using (74) and (79) and rescaling the gauge potential

$$A_{1,2} \rightarrow \hat{G}^{\frac{1}{2}} R^{-1} A_{1,2} , \quad (80)$$

Eq.(78) can be rewritten as

$$\begin{aligned} & \left\{ -\frac{1}{4}(\hat{G}^{-1})^{jk}(\hat{G}^{-1})^{li} + \frac{1}{8}(\hat{G}^{-1})^{ji}(\hat{G}^{-1})^{lk} \right\} F_{ij} F_{kl} \\ &= \frac{1}{2} \sum_{i=1,2} (\partial_0 A_i)^2 + \frac{1}{2} \cdot \frac{R^2}{\hat{G}} \sum_{\alpha,\beta=1,2} (\partial_\alpha A_\beta)^2 . \end{aligned} \quad (81)$$

This tells that the contribution to the one-loop potential from the gauge potential is given by

$$\frac{1}{2} \cdot \frac{R}{(R^4 + h^2)^{\frac{1}{2}}} f_T(1, 1) . \quad (82)$$

Then the total one-loop potential is given by

$$V_T = k(R^4 + h^2)^{\frac{1}{2}} + \frac{d-2}{2} \cdot \frac{R}{(R^4 + h^2)^{\frac{1}{2}}} f_T(1, 1) . \quad (83)$$

When we compare this potential with that of Eq.(19), the only difference is the coefficient in the second term. Therefore the potential (83) is stable and has non-trivial minimum as in Eq.(19).

This potential appears to be unstable since it is unbounded below when $R \rightarrow 0$. However, it would not be true since the non-perturbative effect would play more essential role near $R = 0$. Hence, the quantum effects of abelian gauge field tend to destroy the ground state which exists when D-brane interacts with constant magnetic field.

The one-loop potential in the toroidal membrane ($p = 2$) with the electric background when including the quantum backreaction of the gauge potential can be also found. The kinetic term of the gauge potential is given by

$$\begin{aligned} & \left\{ -\frac{1}{4}(\hat{G}^{-1})^{jk}(\hat{G}^{-1})^{li} + \frac{1}{8}(\hat{G}^{-1})^{ji}(\hat{G}^{-1})^{lk} \right\} F_{ij} F_{kl} \\ &= \frac{1}{2} R^4 \hat{G}^{-1} (\hat{G}^{-1})^{\alpha\beta} F_{0\alpha} F_{0\beta} + \frac{1}{4} (\hat{G}^{-1})^{\alpha\beta} (\hat{G}^{-1})^{\gamma\delta} F_{\alpha\gamma} F_{\beta\delta} . \end{aligned} \quad (84)$$

By choosing the gauge condition (74), we obtain the transverse condition

$$(\hat{G}^{-1})^{\alpha\beta} \partial_\alpha A_\beta = 0 . \quad (85)$$

By using (74) and (85), Eq.(84) can be rewritten as

$$\begin{aligned}
& \left\{ -\frac{1}{4}(\hat{G}^{-1})^{jk}(\hat{G}^{-1})^{li} + \frac{1}{8}(\hat{G}^{-1})^{ji}(\hat{G}^{-1})^{lk} \right\} F_{ij}F_{kl} \\
&= \frac{1}{2}\hat{G}^{-1}R^4(\hat{G}^{-1})^{\alpha\beta}\partial_0 A_\alpha\partial_0 A_\beta \\
&+ \frac{1}{2}(\hat{G}^{-1})^{\alpha\beta}(\hat{G}^{-1})^{\gamma\delta}\partial_\gamma A_\alpha\partial_\delta A_\beta .
\end{aligned} \tag{86}$$

Then the contribution to the one-loop potential from the gauge potential is given by

$$\frac{1}{2} \cdot \frac{(R^2 + e^2)^{\frac{1}{2}}}{R^2} \hat{f}_T(1, 1, -\frac{e^2}{R^2 + e^2}) . \tag{87}$$

Then the total one-loop potential is given by

$$V_T = k(R^4 + 2e^2)^{\frac{1}{2}} + \frac{d-2}{2} \cdot \frac{(R^2 + e^2)^{\frac{1}{2}}}{R^2} \hat{f}_T(1, 1, -\frac{e^2}{R^2 + e^2}) . \tag{88}$$

This potential also appears to be unstable since it is unbounded below when $R \rightarrow 0$.

Similarly, one can find the contribution of gauge fields to static potential. However, in this case such contribution is next-to-leading order of $1/d$ -expansion. Hence, it is not relevant for study of extremum of static potential.

4 Discussion

In the present work we discussed the one-loop and static potentials for toroidal D-brane in constant electromagnetic field. The interesting qualitative result of such study is the fact that classical electromagnetic background may stabilize the one-loop potential. As a result the non-trivial minimum of potential exists (i.e. non-pointlike ground state of D-brane exists).

The properties of static potential (where quantum effects of gauge field are next-to-leading order) are described in detail. It is found that in all cases static potential has stable minimum.

It is not difficult to generalize the results of this study to other backgrounds. For example, one can consider the spherical D-brane in constant electromagnetic field. Here, we found very similar results for one-loop and static potentials as in above case of toroidal D-brane.

Note that we worked in the static approximation. However, it is well-known fact that static approximation is quickly broken in the external electrical field where intensive pairs production occurs. Hence, as extension of this work it would be very interesting to calculate the effective action for D-brane in an external electrical field taking into account also imaginary part of the effective action, i.e. to work beyond static approximation.

Another interesting problem is to study the effective potential for D-brane described by non-abelian DBI action in the background gauge fields. We hope to return to this problem in future research.

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